Non-smooth and level-resolved dynamics illustrated with a periodically driven tight binding model

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Energy pumping into a sea

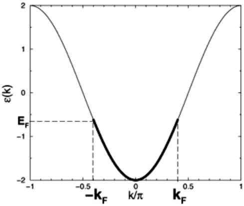
A water wave generator (in an infinite water tank)

How much energy is pumped into the water (per unit time)?

Energy pumping into a Fermi sea

Consider a one-dimensional tight-binding lattice filled with **non-interacting** spinless fermions,

$$\hat{H}_{TBM} = -\sum_{j=-L}^{L} (\hat{a}_j^\dagger \hat{a}_{j+1} + \hat{a}_{j+1}^\dagger \hat{a}_j).$$

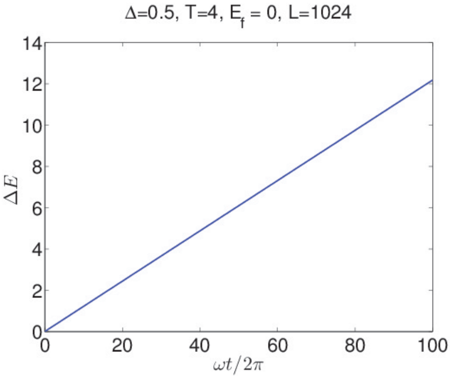


Now start driving the system harmonically:

$$\hat{V}(t) = \Delta \hat{a}_0^\dagger \hat{a}_0 \sin \omega t.$$

How much energy are you dumping into the Fermi sea?

Numerically easy, Analytically not



The slope:

- easy to calculate numerically
- but how to do it analytically?

Reduced to a single-particle problem

A Slater determinant remains as a Slater determinant:

$$\begin{aligned}\Psi(x_1,\ldots,x_N,t) &= \phi_1(t)\wedge\phi_2(t)\wedge\ldots\wedge\phi_N(t),\\ \phi_j(mT) &= \hat{U}^m\phi_j(0),\end{aligned}$$

with the one-period time-evolution operator:

$$\hat{U} = \mathcal{T}e^{-i\int_0^T dtH(t)}.$$

Therefore,

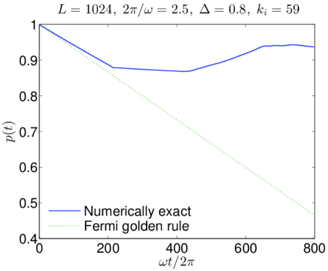
$$\begin{aligned}\Delta E(mT) &= \sum_{j=1}^{N}\langle\phi_j(mT)|H_0|\phi_j(mT)\rangle - \langle\phi_j(0)|H_0|\phi_j(0)\rangle\\ &= \sum_{j=1}^{N}\Delta E_j(mT).\end{aligned}$$

Fermi golden rule?

Fermi golden rule quantitatively insufficient

Consider the transition probability $1 - p(t)$,

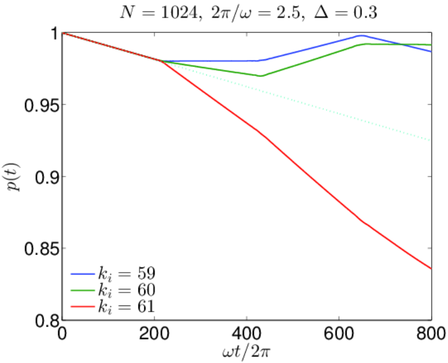
$$p(t) = |\langle \phi(0) | \phi(t) \rangle|^2,$$
$$|\phi(0)\rangle = |k_i\rangle \equiv \frac{1}{\sqrt{N}} \sum_n e^{i2\pi k_i n/N} |n\rangle.$$



The problem has to be solved **non-perturbatively** if the driving amplitude Δ is large.

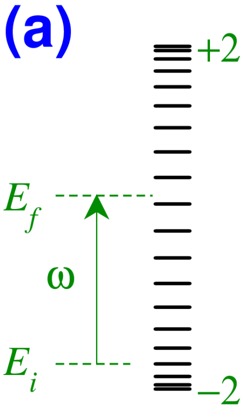
Kinks!

A relatively weak driving (Δ small):



- Kinks appear periodically in time.
- Piece-wise linear!
- Initial level dependence!

Essential features



- Equal spaced (locally)

$$E_{n+1} - E_n = \delta.$$

- Equal coupling

$$|\langle n|\hat{V}|i\rangle| = g.$$

- Weak Δ means it is a first-order effect.

Firsr-order perturbation theory

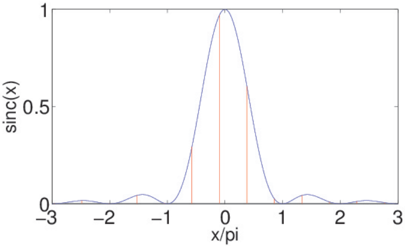
The transition probability is

$$1 - p(t) \quad \simeq \quad 4g^2 \sum_{m\in\mathbb{Z}} \frac{\sin^2[(E_m - E_f)t/2]}{(E_m - E_f)^2} = \left(\frac{4g^2}{\delta^2}\right) W_\alpha(T),$$

with the rescaled, dimensionless time $T \equiv \delta t/2$, and

$$W_\alpha(T) \equiv T^2 \sum_{m\in\mathbb{Z}} \mathrm{sinc}^2[(m-\alpha)T]$$

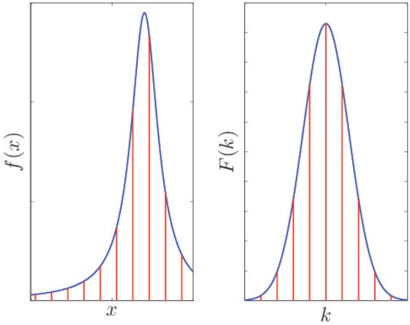
with $\alpha = (E_f - E_n)/(E_{n+1} - E_n)$, $E_n < E_f < E_{n+1}$.



A **uniform** sampling of the $\mathrm{sinc}^2 = \left(\frac{\sin x}{x}\right)^2$ function!

Poisson summation formula I

$$\sum_{m=-\infty}^{+\infty} f(a+mT) = \frac{1}{T}\sum_{m=-\infty}^{+\infty} F\left(\frac{2\pi m}{T}\right)\exp\left(\frac{i2\pi ma}{T}\right).$$



Periodic sampling in **real** space

$\Longleftrightarrow$ Periodic sampling in **momentum** space (simpler?)

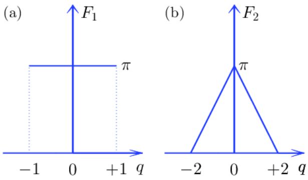
Poisson summation formula II

For our specific problem, we sample the sinc^2 function,

$$W_\alpha(T) \equiv T^2 \sum_{m\in\mathbb{Z}} \mathrm{sinc}^2[(m-\alpha)T]$$
$$= T \sum_{n\in\mathbb{Z}} F_2\left(\frac{2\pi n}{T}\right) \exp\left(-i2\pi n\alpha\right),$$

with

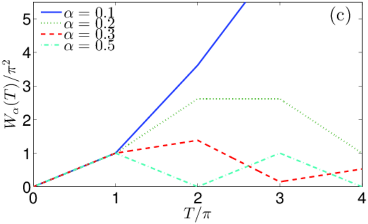
$$F_1(q) \equiv \int_{-\infty}^{+\infty} dx e^{-iqx} \left(\frac{\sin x}{x}\right),$$
$$F_2(q) \equiv \int_{-\infty}^{+\infty} dx e^{-iqx} \left(\frac{\sin x}{x}\right)^2,$$



Both F_1 and F_2 are of **finite** supports!

Only a **finite** number of terms contribute!

A piece-wise linear function of time



- If $0 < T \leq \pi$,

$$W_\alpha(T) = \pi T.$$

  Fermi golden rule! No α dependence!

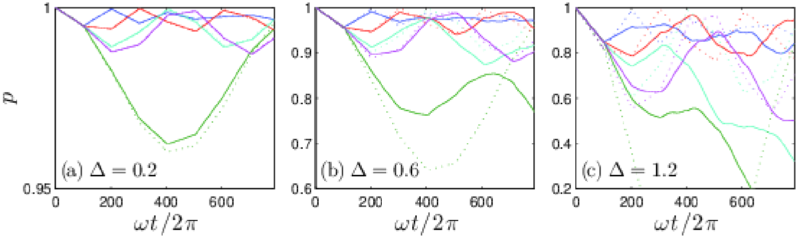
- If $m\pi < T \leq (m+1)\pi$,

$$W_\alpha(T) = \pi T \sum_{n=-m}^{m} \exp(in\theta) - \sum_{n=1}^{m} 2\pi^2 n \cos(n\theta).$$

  The slope is m-dependent and α-dependent!

Beyond the first-order perturbation

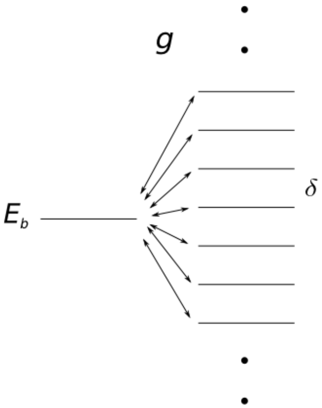
As the driving strength Δ increases:



Five successive states, $k_i = 41, 42, 43, 44, 45$

- Solid lines: numerically exact
- Dotted lines: 1st-order perturbation

An exactly solvable model!



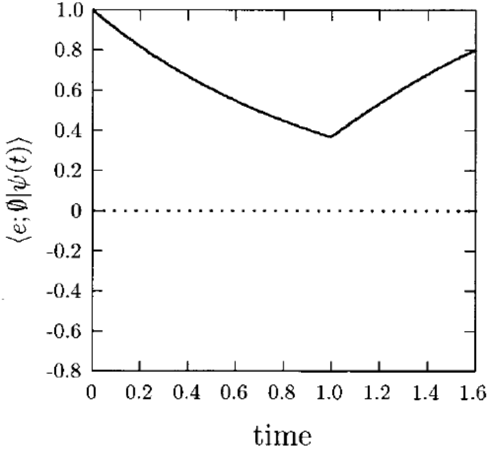
$$
\begin{aligned}
H &= E_b|b\rangle\langle b| + \sum_{n\in\mathbb{Z}} n\delta|n\rangle\langle n| \\
&\quad + g\sum_{n\in\mathbb{Z}} (|b\rangle\langle n| + |n\rangle\langle b|) \\
\psi(0) &= |b\rangle \\
\langle b|\psi(t)\rangle &= ?
\end{aligned}
$$

Realization: a single two-level atom in a multi-mode optical cavity

G. C. Stey and R. W. Gibberd, Physica **60**, 1 (1972).

M. Ligare and R. Oliveri, Am. J. Phys. **70**, 58 (2002).

An exact, non-perturbative result: Kinks are still there!



M. Ligare and R. Oliveri, Am. J. Phys. **70**, 58 (2002).

Summary

- A simple but nontrivial energy pumping (or dumping) problem
  - Not yet unsolved
  - But a closed expression should be possible?
- Non-smooth and level-resolved dynamics
  - Fermi's golden rule can break down even in the first-order perturbation framework
  - Signatures persist beyond the first-order perturbation
- A crystal-clear derivation of the Fermi golden rule
  - No delta function
  - No hand-waving argument
  - A simple fact about the sinc function

JMZ and M. Haque, arxiv:1404.4280. (rejected by PRA)